Your Signature ———————————————

This is a closed book exam. Show all your work. Correct answers with insufficient or incorrect work will not get any credit. Maximum possible score is 50. There are four questions.

Score

1.	(14)	
2.	(14)	
3.	(14)	
4.	(14)	
5.	(14)	
Total.	(60)	

Extra sheets attached(if any):———————————————

Your Signature _____

(1) Find the linearly independent solutions to

$$\frac{d^2x}{dt^2}(t) + 4\frac{dx}{dt^2}(t) + 4x(t) = 0,$$

for $t > 0$.

(2) Find the general solution to

$$\frac{dx}{dt}(t) + ax(t) = b\exp(-\lambda t); \ t > 0$$

where $\lambda > 0$, a, b are real numbers.

(3) Find the Hermite polynomial that is a solution to the equation

$$\frac{d^2x}{dt^2}(t) - 2t\frac{dx}{dt^2}(t) + 6x(t) = 0$$

(4) Show that the following initial value problem does not have a unique solution.

$$\frac{dx}{dt}(t) = 3(x(t))^{\frac{2}{3}}, \ x(0) = 0.$$

(5) Assume that a particular type of bacteria is in an environment where there is no obstacle to growth. Suppose that the population grows at a rate proportional to the current population. Find the time required for the population to double in size.